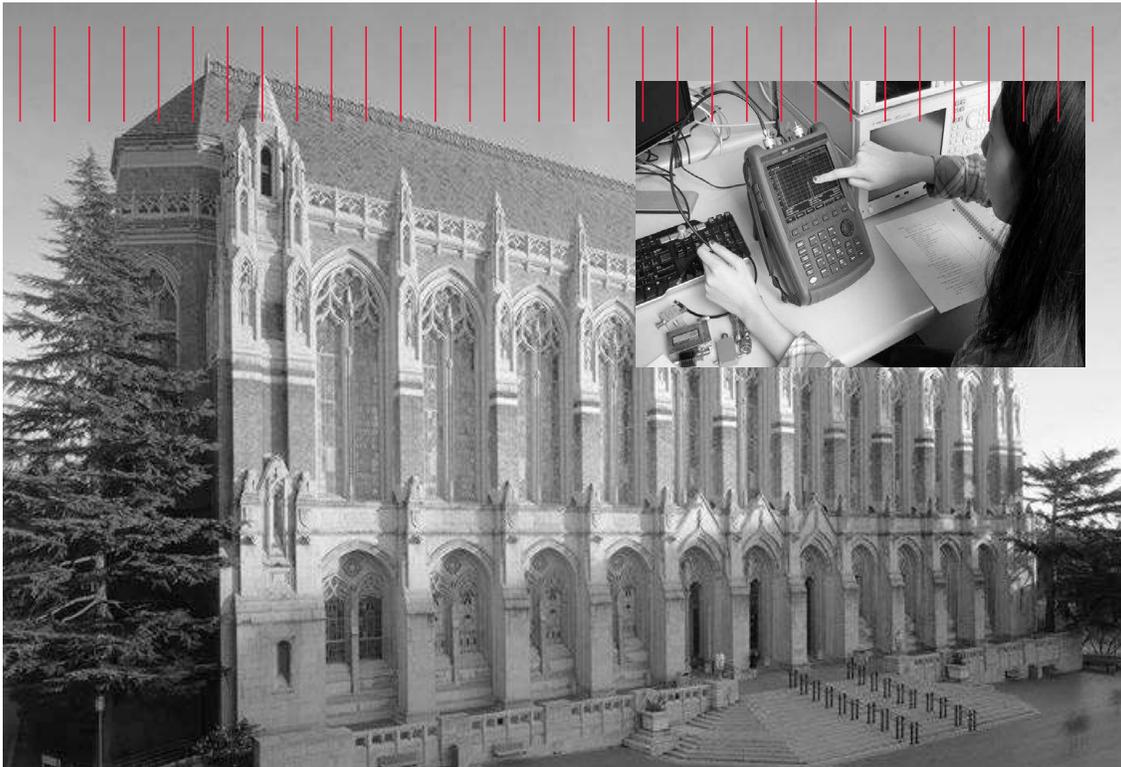


Keysight Technologies

# Spectrum Analysis and the Frequency Domain

University Engineering Lab Series - Lab 2

Application Note





## Introduction

Signals can be represented in either the time or frequency domain. Mathematically, the two representations are connected by either Fourier or Laplace transforms. Signals can be measured in the laboratory in either domain, but using different instruments. While the oscilloscope is the most common instrument for time domain measurements, the spectrum analyzer is the most common instrument for frequency domain measurements. Linear systems can likewise be characterized by either their time domain impulse response or by their frequency domain transfer function. The transfer function proves much easier in many circumstances because input signals are simply multiplied by the transfer function to produce the output frequency domain representation, whereas time domain input signals must be convolved with the impulse response to produce the output time domain waveform. Spectrum analyzers allow the frequency domain representation of a signal to be directly viewed, and network analyzers provide direct measurement of the transfer function in the frequency domain. In this laboratory, the spectrum analyzer will be introduced and its basic operation, functions, and measurements will be developed.

## A quick overview

There are two fundamentally different ways to obtain the frequency domain representation of a signal. The first method is computational in which the signal is first acquired in the time domain and this time domain signal is then transformed into its corresponding frequency domain representation by means of a Fast-Fourier Transform or FFT algorithm. The second method is to directly measure the output amplitude as the signal is fed through a fairly narrow band pass filter which is swept across the frequency span of interest.

The first method is usually referred to as digital signal analysis (DSA) in reference to the computational part that must be performed after the signal has been fully acquired in the time domain. In order to capture the full frequency spectrum of a signal for subsequent computation, the input acquisition must sample at twice or more of the highest frequency component of the signal, the Nyquist rate. For low frequency signals in the audio range with a signal bandwidth of 20 Hz to 20 kHz, the Nyquist rate would be only 40 kS/s, which is easily achieved with modern analog-digital converters (ADCs). Digital signal analyzers or dynamic signal analyzers, both known as DSAs, operate in this manner. They acquire the input signal with a fast input sampler and ADC, store the acquired signal into local memory, compute the FFT, and display the result as the frequency spectrum of the signal. DSAs are designed to run continuously, so that the displayed frequency spectrum lags only a fraction of a second behind the input signal, providing a nearly real-time analysis.

Higher frequency signals can also be captured by a fast digital oscilloscope and stored into memory. Fast oscilloscopes can acquire repetitive signals of up to 500 MHz or more, and many models provide soft key functions to compute and display the FFT of the acquired signal. These FFT computations do not run continuously, but rather take the form of post-acquisition analysis of a given waveform, similar in many ways to the measurement function suite of most modern digitizing oscilloscopes. Examining the frequency spectrum of signals higher than a few hundred MHz becomes difficult because of the limitations on input sampling rate and computational throughput to produce a fast responding displayed output.

DSAs or oscilloscope FFTs do not have the frequency capability to perform signal analysis in the RF and microwave ranges. For this, traditionally a swept frequency analyzer or swept filter analyzer (SFA) is used. At frequencies of several hundred MHz and higher, there isn't sufficient time to perform an acquisition or computation on the signal, so instead a narrow band pass filter is swept across the frequency range of interest and the output from that filter is used to directly measure the frequency spectrum of the input signal. Although the center frequency of the filter may be quite high, the speed of the frequency sweep is much slower, and this allows the output from the swept filter to be sampled, digitized, stored, and displayed, much like what an oscilloscope would do for a lower frequency time domain signal.

While it is possible to create exceptionally sharp band pass filters for signal analysis, it is extremely difficult to sweep them across a range of frequencies without significantly changing their band pass characteristics. The filtering part of an SFA is straightforward; the problem lies with the sweeping. It turns out that constructing an oscillator whose output frequency can be swept linearly over a range of frequencies

is quite practical. A voltage-controlled oscillator or VCO is a sine wave oscillator whose frequency can be tuned by an applied DC voltage. Driving the VCO from a ramp or sweep generator produces an output which starts at one frequency, increases linearly with time up to another end point frequency, and then repeats. The combination of a VCO and a sweep generator is usually known as a sweeper for short. Sweeping an oscillator turns out to be significantly easier than sweeping a filter.

So how does that help solve the problem of spectrum analysis at RF frequencies? An important principle, expressible as a law of trigonometry, details the result of multiplying two sinusoids:

$$\cos(\omega_1 t) \cos(\omega_2 t) = \frac{1}{2} \cos((\omega_1 - \omega_2)t) + \frac{1}{2} \cos((\omega_1 + \omega_2)t)$$

$$\sin(\omega_1 t) \sin(\omega_2 t) = \frac{1}{2} \cos((\omega_1 - \omega_2)t) - \frac{1}{2} \cos((\omega_1 + \omega_2)t)$$

The multiplication of two sinusoids of different frequencies  $\omega_1$  and  $\omega_2$  will produce an output which contains the difference and the sum of those frequencies,  $\omega_1 - \omega_2$  and  $\omega_1 + \omega_2$ . A mixer is a nonlinear electronic device which can produce this desired multiplication of two signals. A mixer combined with a fixed oscillator input, often referred to as the local oscillator (LO) signal, can be used to move an input signal up or down in frequency by an amount equal to the frequency of the LO. Extracting only the sum or difference frequency by means of a filter is known as heterodyning and is a fundamental system concept to many types of radio receivers. Now if the local oscillator instead becomes a swept oscillator, the output from the mixer will be the input signal spectrum moving up and down in frequency by an offset equal to the LO frequency. If a narrow band-pass filter is applied to the output of the mixer, then as the input signal spectrum is moved up and down through the frequency sweep, the band-pass filter will pick out only those components which align to its pass band, performing the same function as if the band-pass filter itself were being swept.

Figure 1 illustrates the essential blocks of an SFA. The conventional names for the signals are: RF, the radio frequency input; IF, the intermediate frequency output from the mixer;  $f_{IF}$ , the filtered IF whose strength represents the frequency content of the input signal at this frequency selected by the LO; LO, the local (voltage controlled) oscillator; and SWP, the voltage sweep signal, most commonly a ramp or saw tooth waveform from the sweep generator.

As an example of how this works, suppose that the input signal was a sinusoidal carrier at 2.0 GHz with AM (amplitude) modulation of 100 MHz. This would give the frequency spectrum of the input a main carrier peak at 2.0 GHz with two smaller sidebands at 1.9 GHz and 2.1 GHz as a result of the modulation. The sweep generator produces a ramped voltage which controls the local oscillator

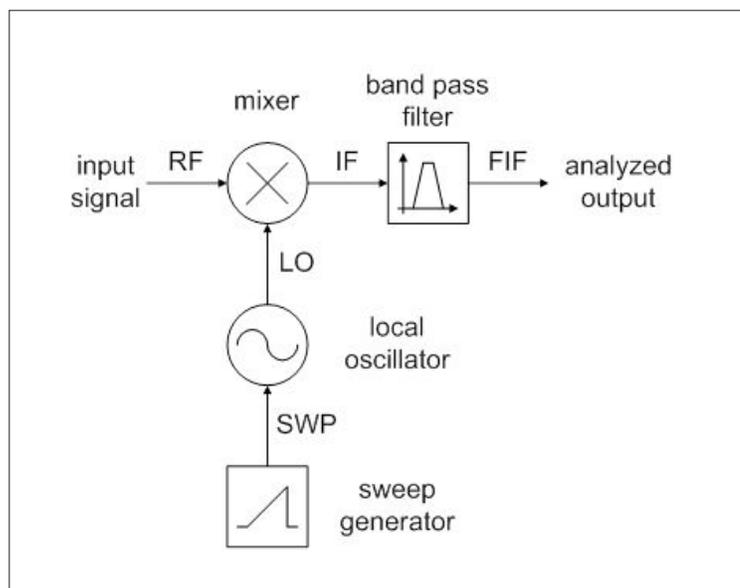


Figure 1. Principal elements of a swept frequency analyzer (SFA)

and sweeps it over a range of 1.0 GHz to 2.0 GHz. The band-pass filter is a fixed tuned filter centered at 500 MHz with a bandwidth of 10 MHz, a Q of 50. When the sweep starts with the LO at 1.0 GHz, the mixer will produce sum and difference frequencies with the RF input. This will shift the 2.0 GHz RF carrier down to 1.0 GHz and also up to 3.0 GHz. The modulation sidebands will track along with the carrier. At this point, nothing lines up with the band-pass filter selected range of 500 MHz so there is no output. At the other extreme, when the local oscillator has swept up to 2.0 GHz, this now shifts the 2.0 GHz RF carrier down to DC and also up to 4.0 GHz. Again, nothing appears at the 500 MHz window of the band-pass filter. However, when the sweep is midway with the local oscillator at 1.5 GHz, the 2.0 GHz RF carrier is shifted down to 500 MHz and up to 3.5 GHz. Now the down shifted RF carrier aligns with the 500 MHz selection of the band-pass filter which will output this IF frequency in proportion to the amplitude of the input RF carrier. This is illustrated in figure 2. When the local oscillator is at 1.4 GHz, the lower modulation sideband of the RF input is aligned with the band-pass filter, producing an output, and similarly, when the local oscillator is at 1.6 GHz, the upper sideband of the RF input is aligned with the band-pass filter, producing another output. Notice that as the local oscillator sweeps up, the band-pass filter in effect scans upward through the input RF signal spectrum.

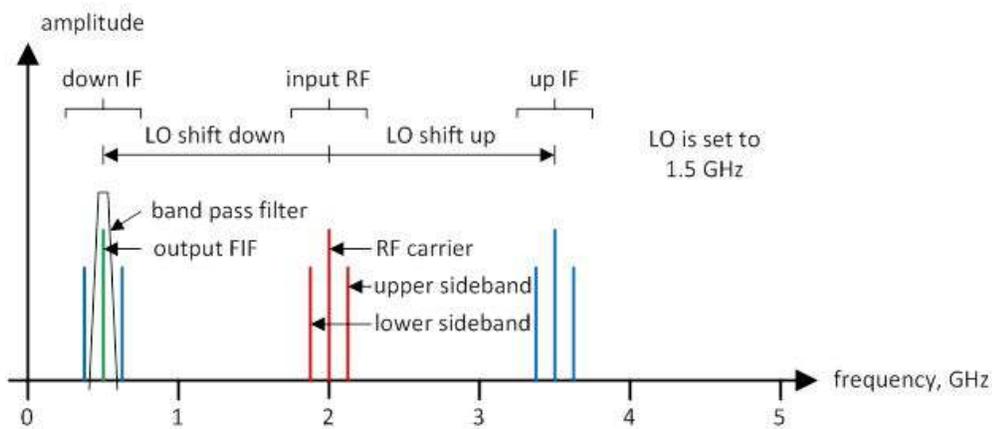


Figure 2. Mixing the RF with the LO to produce an IF which is aligned to the BPF

The filtered IF that passes through the band-pass filter is a sinusoid at 500 MHz. The amplitude of this sinusoid, when matched up to the known LO frequency that is established by the sweep generator, gives the amount of signal present in the RF input at a frequency of  $f_{\text{BPF}} + f_{\text{LO}}$ . Each sweep of the sweep generator thereby creates an  $f_{\text{IF}}$  output whose amplitude traces out the frequency spectrum of the RF input.

The amplitude of the 500 MHz  $f_{\text{IF}}$  output must be converted to a lower frequency voltage level. This is accomplished by an envelope detector, a semiconductor diode in its simplest form, which is followed by a low-pass filter. This low-pass filter is sometimes known as the video filter, because it sets the speed at which the spectrum can be swept through and have the display output accurately follow it. Two important concepts for spectrum analyzers are the resolution bandwidth (RBW) and the video bandwidth (VBW). The resolution bandwidth is effectively the frequency width of the IF band-pass filter, and this determines how closely two

input signal peaks can be spaced and have the spectrum analyzer resolve the difference between them. The video bandwidth is the bandwidth of the low-pass filter that follows the envelope detector, and this determines how fast the sweep can go for the data acquisition and display to accurately follow it. Both of these will be examined in more detail later on.

A complete spectrum analyzer system usually incorporates a few other blocks as well. The input RF signal is usually passed through an adjustable input attenuator and buffer amplifier which are used to adjust the signal level to best optimize the performance of the mixer. Because a spectrum analyzer is normally expected to operate over a rather wide dynamic range of input signal amplitudes, the  $f_{IF}$  output is normally passed through a logarithmically responding amplifier. This naturally makes the output proportional to the input signal strength in units of decibels (dB). Frequency accuracy is normally quite important for spectrum analysis, so the local oscillator is usually derived from an oven-stabilized quartz crystal frequency reference. In some cases it may be digitally synthesized. With many modern analyzers, the reference frequency accuracy can be increased by locking to the GPS signals.

Finally, the video output from the envelope detector is fed into an analog-digital converter to digitize the data, and this data is sent to both digital memory which is used to buffer the displayed output, as well as to store the data for calibration, normalization, and archiving. A more complete block diagram of a spectrum analyzer system is shown in figure 3. In this figure, CRF is the conditioned RF input signal; FR is the frequency reference; and VID is the video output from the envelope detector and low pass filter.

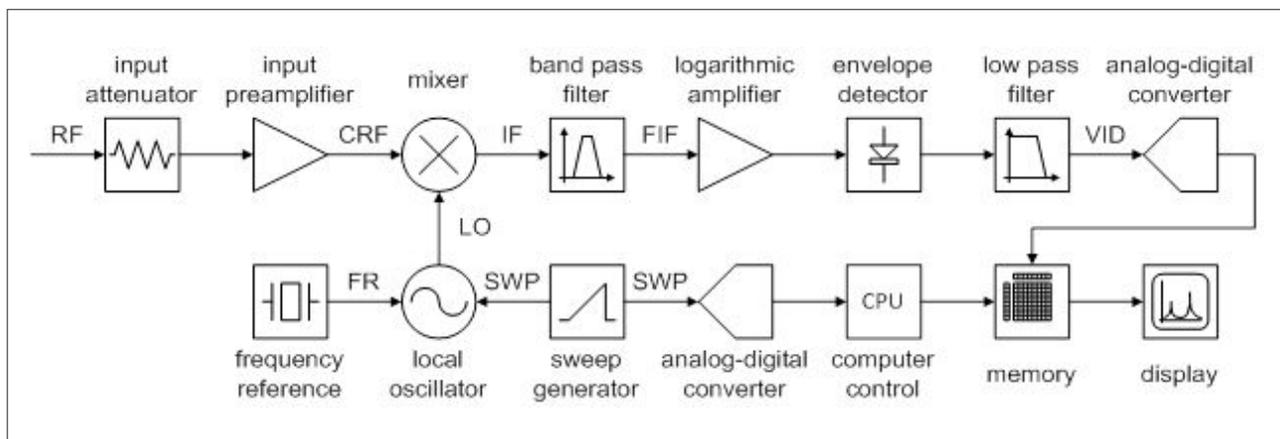


Figure 3. A more complete spectrum analyzer system

Another spectrum analysis method is to use a stepped FFT analyzer, which is the technique used in FieldFox analyzers. FieldFox spectrum analyzers are FFT analyzers that can operate in stepped sweep or stepped FFT mode. Conceptually these analyzers operate similarly to swept analyzers, but the functionality is implemented differently. For example, VBW filtering is accomplished by averaging measurement bins, instead of a low-pass filter. Keysight Technologies, Inc. application note Spectrum Analyzer Basics, publication number 5952-0292, examines the various types of spectrum analyzer architecture.

## Some practice spectrum measurements

The Keysight N9914A FieldFox RF analyzer contains a spectrum analyzer (SA) application which implements essentially the RF system described above. In this laboratory the Keysight N5181A 100 kHz – 3 GHz MXG analog signal generator will be used to provide an input signal for the FieldFox to analyze. This signal generator can in some ways be regarded as a very high frequency version of the standard laboratory function generator, although instead of creating sine, square, triangle, and pulse wave shapes, it creates sinusoidal carriers with various types of applied modulation.

First set up the N9914A FieldFox so that it is running off of the AC power supply. This will insure that the batteries will not run out during the lab. Power up the FieldFox, and wait for the CAT application to load and launch. Attach a Type-N to SMA adapter to port 2 if one is not already present. After the CAT application is running, press the Mode button to bring up the application selection. Press the SA soft key to load and launch the spectrum analyzer application. Note that the spectrum analyzer input needs to go into port 2, and that the spectrum analyzer will start up with the full span sweep of 0 Hz to 6.5 GHz.

Before applying any external signal to an analyzer, regardless of the type, the RF power levels must be considered so that the delicate and expensive front-end amplifier of the analyzer does not get destroyed from excessive power dissipation or over-voltage. The N9914A FieldFox inputs can handle a maximum of +27 dBm, or 0.5 Watts. The maximum output from the N5181A signal generator is +17 dBm, or 50 mW. So in this case, there is no chance for the signal generator to destroy the FieldFox – all is well. But never simply assume this is the case; always check the power levels before running the risk of damaging an expensive instrument.

Next, configure the N5181A signal generator to output a 500 MHz sine wave at a power level of –20 dBm, or 10  $\mu$ W. To do this, first insure that the signal generator has AC line power; if so the amber LED above the power button should be glowing to indicate that the instrument is in a standby mode. Press the power switch to turn the instrument on, and the amber LED should change to a green LED, and the front panel display should light up into its default state. Attach a Type-N to SMA adapter to the output Type-N connector on the front panel if one is not already present.

Then use an 18-inch long SMA coaxial cable to connect the output of the signal generator to the input on port 2 of the FieldFox. To configure the sine wave carrier from the signal generator, first press the FREQ button to obtain the frequency menu. The instrument normally wakes up in this menu as the default. Then press 500 from the keyboard followed by the MHz soft key to the right of the display to set the frequency. Next press the AMPTD button to obtain the amplitude menu. Press –20 on the keyboard followed by the dBm soft key. Next, press the RF On/Off button just above the output connector. The green LED below this button should glow and the display panel should now indicate an output amplitude level of –20.00 dBm. The FieldFox should now show a prominent single peak on its display whose peak shows a power level of –20 dBm.

A single sharp peak in the frequency domain should indicate a perfect sine wave in the time domain, but the FieldFox can reveal quite a bit more detailed information about the signal generator. Increase the output power from the signal generator to 0.0 dBm, or 1.0 mW. An easy way to do this is to first get into the amplitude options by pressing the AMPTD button, and then position the cursor under the units position of the -20.00 dB indicator by using the arrow keys on either side of the Select button. Once the unit's position has been highlighted, simply rotate the control dial to increase or decrease the amplitude in 1.00 dB increments. Now slowly increase the signal generator amplitude from -20.00 dBm upwards to 0.00 dBm and observe that the main peak displayed on the FieldFox also increases in amplitude. As the power level is increased, a secondary peak should start to appear to the right of the main peak. The signal generator output can be toggled to verify that this secondary peak is indeed coming from the signal generator. On some signal generators, the power level may have to be increased beyond 0 dBm to see the secondary peak. If the increase in signal generator power results in an "ADC Over Range" message on the FieldFox, increase the RF attenuation on the FieldFox by pressing the Scale/Amptd key and then RF Atten soft key, and setting that to 20 dB.

Next, the marker function can be used to get some quantitative numbers for these two peaks. Press the Marker button on the FieldFox to bring up the marker menu and turn on the first marker point. The coordinates of the first marker will appear in green letters in the upper right of the display, and the display will also show a solid green diamond at the current marker position. The easiest way to move the marker to the main peak is to simply press the soft key labeled Peak. The marker will jump to the largest peak in the display, and in this case it should have coordinates close to 500.000 MHz and 0.00 dBm. Record the specific values of this main peak. Now use the dial on the FieldFox to move the marker to the top of the secondary peak. The coordinates for this should be around 1.000 GHz and -45 dBm. Similarly, record the values of this secondary peak. The high frequency accuracy of the spectrum analyzer shows that the frequency of the secondary peak is exactly twice that of the primary peak; hence, this is the second harmonic of the first peak. Instead of creating a perfect sine wave, the signal generator is producing one with just a little bit of second harmonic distortion.

As the power level of the signal generator is varied, it should have been observed that the amplitude of the secondary peak tracks that of the first. Rather than giving the secondary peak an absolute power level, it makes more sense to specify its power level relative to the main carrier peak. From the previous measurements, the secondary peak remains at a level of about 45 dB below that of the carrier. The second harmonic peak could then be stated to have an amplitude of -45 dBc, which simply means 45 dB below the carrier. Compute the signal level of the second harmonic from the previously recorded readings, and express this in dBc. 45 dB is a factor of 31,600 in power, or 177 in voltage, so the second harmonic is actually quite small in comparison to the carrier. The figure of -45 dBc second harmonic distortion is actually pretty good for a signal generator. The wide dynamic range of the FieldFox spectrum analyzer is what allows this measurement to be made. This same measurement would be nearly impossible to make in the time domain using an oscilloscope. Notice that the output of the signal generator needed to be increased to 0.00 dBm in order to get the second harmonic peak to emerge from the noise level.

Next the effects of frequency span and resolution bandwidth on the displayed noise level will be examined. Adjust the FieldFox frequency sweep to a center frequency of 500 MHz with a frequency span of 500 MHz. At the bottom of the FieldFox display it should indicate a resolution bandwidth (Res BW or RBW) of 3.00 MHz and video bandwidth (VBW) of also 3.00 MHz. The sweep time should show about 100 ms for the 401 display points, indicating about a 10 Hz refresh rate on the spectrum analyzer display. Recall that the resolution bandwidth is set by the IF band-pass filter and the video bandwidth is set by the low-pass filter that follows the envelope detector.

When the frequency span of the FieldFox is changed, the control software (known as firmware) automatically determines the best RBW and VBW filter cut offs to use. On older style spectrum analyzers, these would have to be adjusted manually. At the current span of 500 MHz, the displayed average noise level (DANL) should be seen to be around  $-70$  dBm. At the prior full frequency span of 30 kHz – 6.5 GHz, roughly a factor of 10 wider, the DANL was around  $-60$  dBm. Now change the span to 50 MHz. The carrier peak should look about the same, but the DANL should have fallen to about  $-80$  dBm. The RBW and VBW should also have been reduced to 300.0 kHz. At this span of 50 MHz, the 401 points in the display gives 125 kHz between points. Comparing that to the 300 kHz RBW means that the display is not quite three times oversampled. Experiment a little further by reducing the span to 5.0 MHz which should produce a RBW of 30.0 kHz and a DANL of  $-90$  dBm. Keep going until something changes, and make a note of what that is. The key point to observe is how frequency span, RBW, and DANL are interrelated. This is a basic tradeoff in all swept frequency analyzer instruments.

Next amplitude modulation (AM) will be examined. Return the FieldFox to a center frequency of 500 MHz and a span of 500 MHz. Return the N5181A signal generator to a carrier of 500 MHz with an amplitude of  $-20$  dBm<sup>1</sup>. Press the AM menu button on the signal generator to bring up the amplitude modulation options. Using the soft keys to the right of the display panel, set the AM Type to EXP, set the AM modulation rate to 1.00 MHz, and set the modulation depth to its maximum value of 40.00 dB. Press the uppermost soft key to turn the AM On, and then insure that the modulation output is enabled by pressing the Mod On/Off button above the output connector. Its green LED should be glowing along with the one beside it to indicate the carrier output or RF is on.

One may have been expecting to see some change in the FieldFox display when the modulation was turned on; however, the display remains unchanged. The reason is that the AM modulation of 1.00 MHz would only introduce sidebands of 1.00 MHz away from the carrier, and the current span setting of 500 MHz produces a resolution bandwidth of 3.00 MHz – too wide to pick up these AM sidebands. Thus, this particular span and sweep setting cannot resolve the AM the carrier peak. Reduce the span further to 5.0 MHz, and the two AM sidebands should now become quite distinct, each located exactly two divisions to either side of the carrier peak. Notice that the AM sideband peaks are also rather small in amplitude in comparison to the carrier. In order to detect these AM sidebands, the reduction in frequency span was necessary to not only reduce the resolution bandwidth, but also to reduce the displayed average noise level to where the sideband peaks would emerge. Capture a screen image of this AM modulated carrier. This will be a good reference for later on.

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1. The amplitude modulation exercise is based on Keysight's "A" model signal generators; that is, signal generators where the model number ends in the letter A such as the N5181A. This section is not applicable to "B" model signal generators such as the N5172B.

One reason that the AM sidebands are so small is because the AM modulation rate is rather high at 1.00 MHz. Reduce the AM modulation rate to 10 kHz, now a high pitched squeal within the audio range, and set the AM Type to LIN and the modulation level to 100% using the soft keys of the signal generator. Reduce the span on the FieldFox to 500 kHz and notice now that the carrier appears to have many sidebands to either side of it. Each matching pair of these corresponds to different harmonics of the modulation frequency of 10 kHz. Change the span to 50 kHz and the display should now show five clear peaks: the center carrier at 500.000 MHz and  $-20.00$  dBm, the first harmonic pair at  $500.000 \text{ MHz} \pm 10 \text{ kHz}$  and about  $-26.00$  dBm, and the second harmonic pair at  $500.00 \text{ MHz} \pm 20 \text{ kHz}$  and about  $-45$  dBm. Note carefully that this second harmonic distortion arises from the modulator section of the signal generator, and it is distinct from the second harmonic distortion that was first seen from the signal generator's carrier oscillator alone. Use the knob on the signal generator to vary the AM modulation rate in 1.00 kHz increments. Observe that the separation between each of the adjacent peaks is equal to the AM modulation rate.

Next FM modulation will be examined. Turn the AM modulation off by pressing the upper soft key of the signal generator to set AM to Off. Also press the Mod On/Off button above the output connector to disable the modulation for the time being. It is always good practice to disable the output of a generator before making new adjustments. Once the adjustments are made, then turn the output back on. Press the FM/ $\Phi$ M menu button to bring up the frequency modulation options of the signal generator. Use the soft keys to set FM mode, FM to On, FM deviation to 10 kHz, and FM rate to 1.0 kHz. Press the Mod On/Off button above the output connector to enable the modulation output. On the FieldFox, set the center frequency to 500 MHz and the span to 50 kHz. There should now appear many peaks in the display. Capture a screen image of the FM modulated carrier for later comparison. Use the knob on the signal generator to change the FM modulation rate from 1.0 kHz to 10.0 kHz in 1.0 kHz increments. Notice that the separation between the adjacent peaks is again equal to this modulation rate, just as in the case of AM. Return the FM modulation rate back to 1.0 kHz.

Use the knob on the signal generator to change the FM deviation from 1.0 kHz up to 100 kHz in 1.0 kHz increments. Change the FieldFox span to 500 kHz in order to observe the full spectrum of the output. The overall width of the FM spectrum is approximately twice the FM deviation. Keep in mind that the signal generator's carrier is still set to an amplitude of  $-20$  dBm. When the FM deviation is small, the carrier is clearly present at close to its prescribed amplitude of  $-20$  dBm. However, when the FM deviation becomes large, the carrier is effectively split into many smaller peaks, each with significantly smaller amplitude, roughly in the range of  $-30$  dBm to  $-40$  dBm.

Frequency modulation is often categorized as narrow or wide deviation, or sometimes narrow or wide band, depending upon how the FM modulation rate compares to the FM deviation. Narrow deviation FM is where  $f_{mod} > f_{dev}$ , and the FM spectrum appears similar to AM with a distinct and strong carrier. Wide deviation FM is where  $f_{mod} < f_{dev}$  and the FM spectrum gets spread out into many peaks without any prominent carrier.

## Some further analysis

Whereas wide deviation FM can usually be recognized by its larger frequency spread and its lack of a prominent carrier, it often becomes more difficult to distinguish between AM and narrow deviation FM. Oftentimes the application area provides a clue. Commercial FM radio broadcast stations are usually wide deviation FM with a deviation of around 75 kHz which gives them a higher signal-to-noise ratio (SNR) than an AM system of the same average power. Short range personal radios operating in the FRS (family radio service) band are usually narrow deviation FM with a deviation of only a few kHz. Knowing the FM deviation can help in identifying various signals.

Both AM and FM can be expressed in fairly simple mathematical terms for sinusoidal carriers with single tone modulation. The sinusoidal carrier can be written as

$$c(t) = C \cos(\omega_c t)$$

and the modulation tone can be written as

$$m(t) = M \cos(\omega_m t)$$

The output of an AM system would simply be

$$y_{AM}(t) = [1 + m(t)]c(t)$$

using the same trigonometric relations as presented earlier can be expressed as

$$y_{AM}(t) = C \cos(\omega_c t) + \frac{1}{2} CM \cos((\omega_c - \omega_m)t) + \frac{1}{2} CM \cos((\omega_c + \omega_m)t)$$

This expression directly gives the observed AM frequency spectrum with the first term describing the carrier, the second term the lower modulation sideband, and the third term the upper modulation sideband. Suppressed carrier AM systems are those which only transmit the sidebands and eliminate the unity term which reproduces the carrier. Such suppressed carrier AM systems are considerably more power efficient, although more challenging to build a receiver for. Reduced carrier AM systems compromise by simply reducing or lowering the amplitude of the carrier. Since the information content in the sidebands can be considered redundant, single sideband (SSB) AM systems filter off one or the other of these to create a slightly more power and frequency spectrum efficient transmission.

AM systems do have one limitation which is the depth of the modulation. The amplitude  $M$  should not exceed unity, because otherwise the term in square brackets could become negative, producing phase inversion of the carrier. This leads to a variety of distortion problems and also loss of carrier phase lock. The depth of AM modulation is usually expressed in terms of the modulation index which varies between 0 and 1.

The factor  $M$  above is equivalent to this modulation index. FM systems get around this limitation because there is no such hard limit on the depth of frequency modulation, and in fact extremely large FM modulation depths can be used to increase the effective signal-to-noise ratio (SNR).

In an FM system, the mathematics is a bit more complex. Here, the frequency  $\omega_c$  of the carrier is modulated instead of the amplitude  $C$ . The argument of the cosine is the integral of the frequency, also known as the phase. Frequency modulation can be expressed by

$$\omega = \omega_c + \omega_{\Delta} m(t)$$

where  $\omega_{\Delta}$  is the FM deviation, so that the phase argument of the cosine carrier then becomes

$$\omega t = \omega_c t + \int^t \omega_{\Delta} m(t) dt$$

For a sinusoidal carrier and a single tone modulation, the FM signal then becomes

$$y_{FM}(t) = C \cos \left[ \omega_c t + M \frac{\omega_{\Delta}}{\omega_m} \sin(\omega_m t) \right]$$

Instead of creating the multiplication of two trigonometric functions as in AM, FM creates a trigonometric function which is the argument of another, a considerably more complex relationship. The FM signal can still be decomposed into its frequency spectrum representation, although this now involves Bessel functions of the first kind,  $J_k(x)$

$$y_{FM}(t) = C J_0 \left( M \frac{\omega_{\Delta}}{\omega_m} \right) \cos(\omega_c t) + C \sum_{k=1}^{\infty} J_k \left( M \frac{\omega_{\Delta}}{\omega_m} \right) \left[ \cos((\omega_c + k\omega_m)t) + (-1)^k \cos((\omega_c - k\omega_m)t) \right]$$

Of first note is that this is an infinite series, so there are truly an infinite number of sidebands for FM. However, the Bessel functions do fall off with increasing argument, although in an oscillatory manner and in practice only a few of the terms in the infinite series remain with a significant contribution. The key parameter in how many terms remain significant is the ratio  $M\omega_{\Delta}/\omega_m$ , a measure of the FM deviation, relative to the modulation rate. A rough rule of thumb is that the effective bandwidth of an FM system is

$$BW_{FM} \approx 2(f_{\Delta} + f_m)$$

This was what should have been observed in the previous FM spectrum measurements.

## Measurement assignment

Because of their wide dynamic range and superb frequency accuracy, spectrum analyzers are the premier instrument for electronic snooping of radio signals and other wireless transmissions. Because of its portability, FieldFox is an exceptionally able instrument for signal snooping.

First examine the commercial United States FM broadcast band of 87.5 MHz – 108.0 MHz, and investigate a few additional features of the FieldFox spectrum analyzer. For this, a suitable antenna must be connected to port 2 of the FieldFox. Disconnect the FieldFox from the signal generator; the signal generator can be shut down as it will not be needed further for this lab. Remove the Type-N to SMA adapter and replace it with a Type-N to BNC adapter. Connect a telescopic whip antenna to the BNC connector and extend the antenna to its full length of about 19 inches. By adjusting the length, most telescopic whip antennas can be used for reception over a range of 25 MHz to 1300 MHz.

Set up the FieldFox for its spectrum analyzer application and set the frequency sweep to run from 87.5 MHz to 108.0 MHz. Alternatively, 90 MHz to 110 MHz also works well and aligns the display divisions to even MHz multiples. At this point, several peaks should be visible in the display, typically at power levels of around  $-50$  dBm to  $-30$  dBm. These are the local commercial FM radio stations.

From the frequency sweep menu, press Measure (key 1), which brings up some very useful options for the spectrum analyzer. Press Radio Standard, the left most soft key, and then press Radio Standard (None), to show a vertical list of predefined radio standard frequency bands. Use the dial to select the desired one, followed by the Enter key. For the current purposes, the frequency band has already been set, so simply select None at the top of the list, and Enter. Press the Back button to return to the Measurement options. Different radio standards can also be loaded into FieldFox to make specific measurement set ups easier to do and to avoid having to remember different frequency bands.

Next, press the Tune & Listen soft key, and then the FM Wide soft key. This turns on the wide deviation FM demodulator and routes the output to the built-in audio subsystem and speaker on the FieldFox. A white vertical marker bar should appear on the display. Press the Tune Freq soft key and use the dial to move the white marker bar on to one of the signal peaks. The FM radio station should be able to be heard through the FieldFox speaker. The default listen time is fairly short at 2.5 seconds, so press the Listen Time soft key and rotate the dial to increase this to a larger value to keep the sound from being segmented. Volume can similarly be controlled using the Volume soft key. Experiment with this feature of the FieldFox spectrum analyzer and record the frequencies and power levels of some of the FM stations which can be received. The marker function is very handy for doing this.

Use the FieldFox to scan through the commercial AM radio band from 540 kHz to 1610 kHz. Note that the telescopic whip antenna will not be a good match for these low frequencies, but some reception might still be possible.

Experiment to see if the FieldFox can pick up cell phone transmission. The specific frequency will depend upon the phone type and the service provider. Some common frequencies to try would be 800 MHz, 850 MHz, 1700 MHz, 1900 MHz, or 2100 MHz. AT&T and Verizon both use 850 MHz for voice. Make a note of the power levels and signal bandwidth that is being used by these transmissions. For reception in the 800 MHz to 900 MHz range, the short, flexible BNC antenna may work best. Since cell phone transmissions are usually brief, using the Run/Hold button can be useful to temporarily capture a signal so that it can be measured later with the marker after the transmission has ended.

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*This application note was created by Professor Bruce Darling, from University of Washington's Electrical Engineering Department, in collaboration with Keysight Technologies' handheld team within the Component Test Division. The content is designed to complement an introductory course in undergraduate electromagnetics.*

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